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Dynamic Symmetry Breaking in Physics of Mesons and Top Condensation Models

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ABSTRACT

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The dissertation was performed at N. N. Bogoliubov Laboratory of Theoretical Physics (BLTP) of the Joint Institute for Nuclear Research.

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Scientific Secretary of the Dissertation Council (Technical Secretary),

PhD in Physics Bystritsky Yury Mikhailovich

General description of the work
Relevance of the topic

Vacuum is a foundation of the world. The excitations of the vacuum are the observed elementary particles. The Universe is built from them. It is for this reason that the study of the vacuum and its properties is one of the most fundamental tasks of modern theoretical physics. In particular, it is one of the central problems of the strong interaction physics.

The ground state of quantum chromodynamics (QCD) cannot be the conventional Fock vacuum empty of particles and fields. The QCD vacuum with zero field strength is unstable, and decays into a state with a calculable non vanishing value of the scalar field (condensate) [1]. However, on some more deep level, a homogeneous vacuum field is also unstable [2]. Nonetheless, the scalar condensates give an effective long-distance description of the vacuum [3]–[5], and at short distances, below the QCD scale $\Lambda_{QCD} \sim 0.2\, GeV$, the vacuum may have structure. What are characteristic properties of the QCD vacuum?

The excitations of the QCD vacuum – mesons, baryons, glueballs – are color singlet states. The spectrum has no any resemblance to a field content of the QCD Lagrangian. Color confinement of quarks and gluons at scale $\Lambda_{conf} \sim \Lambda_{QCD}$ makes colored degrees of freedom unobservable. According to S. Mandelstam, Y. Nambu, and G. 't Hooft [6]–[8], non-Abelian chromomagnetic monopoles condense into dual Cooper pairs, resulting in the formation of non-Abelian chromoelectric flux tubes (flaxons) between color charges. Qualitatively, color confinement in QCD can be understood as a result of the dual to the Meissner effect. Since dual superconductor models explain confinement of quarks in terms of an electromagnetic dual theory of superconductivity, it can be assumed that the QCD vacuum has the property of a superconductor.

Spontaneous chiral symmetry breaking (χSB) is another important property of QCD vacuum. Although its scale $\Lambda_{\chi\mathrm{SB}} \sim 4\pi f_\pi \sim 1\ GeV$ differs from Λ_{conf}, it is not excluded that both phenomena are interrelated, because perturbative calculations are generally limited to reactions involving a scale of at least 1 GeV. This is the case, for example, in a picture developed in [9], [10], where it has been argued that χSB is an inevitable concomitant of confinement. If this is true, then χSB should also be related to the superconductivity property of the QCD vacuum.

The pioneering works in this direction belong to V. G. Vaks, A. I. Larkin [11], and independently to Y. Nambu and G. Jona-Lasinio, who suggested a dynamical model of elementary particles based on an analogy with superconductivity [12], [13]. This model was inspired by the success of the Bardeen- Cooper-Schrieffer (BCS) theory [14], [15] in the description of spontaneous gauge symmetry breaking in superconductors, and the concept of quasiparticles as fermionic excitations in the BCS medium independently introduced by N. N. Bogoliubov [16], [17] and G. Valatin [18]. The method of quasiparticles together with the Bogoliubov transformation proved to be effective for constructing a theory of superconductivity for both the initial electron-phonon Hamiltonian and the BCS model Hamiltonian with direct electron interaction. Bogoliubov et al. [19] proved that the results obtained for the model BCS Hamiltonian with factorized interaction are asymptotically exact in the thermodynamic limit $V \to \infty$. This proved the validity of using the self-consistent mean field method in the theory of superconductivity. Although we do not have such proof in the NJL model [20], the appearance of a nonzero vacuum expectation value of the scalar field does not contradict the long-distance behavior of the hadronic vacuum.

Various versions of the NJL model [21]–[34] are currently actively used to study the phase structure of the QCD vacuum, its thermodynamic properties, and also its behavior in an external electromagnetic field. Such studies are stimulated by modern experimental efforts and allow deeper insight into the essence of the problem of QCD vacuum. One of these areas of research is the problem of the chiral anomaly and its various manifestations in specific physical processes. A theoretical description of anomalous processes in the case when vector and axial-vector meson modes are excited is still an unsolved problem and therefore requires a careful study [35]–[37].

One of motivations of this thesis is to contribute to the systematic study of the odd-intrinsic parity sector of QCD by suggesting a new method for calculations of an impact of spin-1 resonances into three-point and four-point Green functions. We show that using our approach, it is possible to obtain results which fulfil the QCD Ward identities in the presence of the pseudoscalar – axial-vector mixing, i.e., our new method resolves the long-standing problem of πa_1*-mixing contributions to the low-energy odd-parity amplitudes. The method is based on the specific properties of the QCD vacuum considered as a state with the dynamically broken Nambu-Goldstone symmetry.*

The other issue where NJL mechanism of spontaneous symmetry breaking may be useful is related to the existence of light fermionic partners of the top quark. The top condensation models can be used to explore the origin of mass, for instance, the reason behind the greatness mass of top quark compared with other known quarks [38]–[47]. In these models, at high energies $\Lambda \gg \Lambda_{EW} \approx 250\, GeV$, the $SU(2)_L \times U(1)_R$ gauge symmetry group of electroweak interactions is dynamically broken by effective four-quark interactions. Owing to a strong coupling, in the fermion spectrum of the theory, a gap appears (the nonzero mass of the *t*-quark) and, as a consequence, the boson condensate is formed predominantly of the third-generation quarks. The collective excitations of the condensate manifest themselves in the form of boson modes associated with composite (quark–antiquark) Higgs bosons, the dynamics of which at low energies $\mu \ll \Lambda$ is described by an effective action which can be found by integrating out the short-distance components of quark fields at leading $\frac{1}{N_c}$ order, where N_c is the number of the color degrees of freedom of quarks. It is supposed that induced four-quark interactions should explain the origin of the Higgs sector of the Standard Model (SM).

The models that contain two Higgs doublets were studied in [43], [48]–[50]. It was noticed that, with the SM extended in the way proposed in [48], [49], the phenomenological constraint $m_t \gg m_q$, where $q = u, d, s, c, b$, restored the previously known result $m_H = 2m_t$. Is it possible to overcome the above difficulties? Finding a way to solve this problem is another motivation for this thesis.

Another motivation of this thesis is to contribute to the systematic study of the Higgs sector of the SM by studying the spectrum of scalar modes through the Nambu sum rule approach. We are motivated by the hypothesis that a single lone Higgs boson is unlikely to exist – there may be a rich spectrum of Higgs bosons, presenting a new spectroscopy in nature. Thus, the new particles we will consider are exclusively new massive Higgs iso-doublet generated by the dynamically broken Nambu-Goldstone symmetry.

Indeed, understanding flavor physics will likely involve the discovery of new particles, which can be associated with the mystery of the origin of the small parameters of the SM [51]. For instance, the observed Higgs-Yukawa coupling of the b-quark is small, $y_b \simeq 0.024$. This small parameter may have a perturbative origin, arising from virtual effects involving new heavy particles with larger couplings, i.e., a small parameter starts as a large parameter that is subsequently power-law suppressed. There are, of course, many theoretical ways to achieve this. Here we suppose that a new heavy Higgs iso-doublet can be an origin for such suppressions and the corresponding dynamics is described by the four-quark model proposed by Miransky, Tanabashi, and Yamawaki (MTY) [43].

The strategy plan of the work

We explore possible consequences of the dynamic chiral symmetry breaking arising due to the Nambu and Jona-Lasinio (NJL) mechanism [52]. The local four-quark interactions of NJL type are known to be a useful ground to construct the effective Lagrangians describing the dynamics of collective quark-antiquark bound states formed in the strong coupling regime. These Lagrangians suffer from the mixing between the Goldstone and axial-vector modes. A standard diagonalization creates the well-known problem in the description of anomalous processes which includes the electromagnetic interactions of soft pions and leads to the violation of a number of famous low-energy theorems of quantum chromodynamics. In this thesis we suggest the new method to solve this problem. It is based on the careful treatment of surface terms arising in the calculation of anomalous triangle diagrams. In particular, our formalism leads to the deviation from the vector meson dominance hypotheses.

The other aspect of the NJL mechanism is the Nambu sum rules, which relates the spectrum of collective modes with the energy gap in the spectrum of quasi-particle excitations [53], [54]. We calculate for first time the spectrum of spinless modes in the NJL type model with the $SU(2)_L \times U(1)_R$ symmetrical four-quark-interaction proposed by Miransky, Tanabashi, and Yamawaki (MYT) to explain the huge mass of the top-quark in the standard model of electroweak interactions. To this end, the Schwinger-DeWitt approach to the problem has been applied for the first time. We show that the $U(1)_A$ symmetry breaking, for which the't Hooft four-quark interaction is responsible, causes deviation from the standard Nambu sum rule. Nonetheless, we demonstrate that the Nambu sum rule is not violated to the leading $1/N_c$ order. In accord with the latter strategy plan of the work, **the objectives of this thesis** can be put in the following manner:

a) To solve the long-standing πa_1-mixing problem in the anomalous part of the effective meson Lagrangian with electromagnetic interactions.

b) To give theoretical description of the anomalous $f_1 \to \gamma\pi^+\pi^-$, $a_1 \to \gamma\pi^+\pi^-$, $\gamma \to 3\pi$, $\eta \to \gamma\pi^+\pi^-$, and $\eta' \to \gamma\pi^+\pi^-$ decays, which would consistently take into account the mixing between pseudoscalar and axial-vector states.

c) To obtain the effective Lagrangian of the composite two Higgs doublet model on the bases of the Schwinger-DeWitt proper-time method.

d) To establish and investigate the Nambu sum rule in the two-Higgs doublet model of Miransky, Tanabashi, and Yamawaki [43], [44].

Theoretical and practical significance

(a) It is commonly accepted and supported by corresponding calculations that electromagnetic interactions of mesons, after bosonization of the theory described by the local four-quark vertices of the NJL-type, have a specific form which corresponds to the vector meson dominance (VMD) hypothesis; i.e., photon interactions with charged hadrons are fully mediated by the ρ, ω, and ϕ neutral vector mesons. Here, we demonstrate that this apparently self-consistent picture is violated in the anomalous processes. The reason for that is usually neglected and treated as irrelevant. It concerns the procedure of removing the mixing between the unphysical axial-vector field a'_μ and pseudoscalar mesons [27], [55], [56]. The transition to the physical axial-vector state a_μ is usually carried out through a linear change of variables $a'_\mu = a_\mu + km\partial_\mu\pi$, where m is the constituent-quark mass and the dimensional constant k is expressed in terms of the mass of the axial-vector meson $k = 3/m_{a_1}^2$. This standard procedure involves a derivative. In the presence of electromagnetic interactions, as we have shown for first time, this derivative should be replaced by the gauge covariant one because, otherwise, the replacement violates the gauge invariance of the effective meson-photon Lagrangian. We show that this modification does not affect a real part of the effective action, but is of great importance to the imaginary anomalous part of the action. Substituting the gauge covariant derivative gives rise to essentially new contributions arising due to surface terms of anomalous triangle quark diagrams. It is these terms that make it possible to ensure the fulfillment of Ward identities, which would otherwise be violated. On the other hand, as it is shown in the thesis, new contributions lead to a deviation from the VMD picture which usually holds in the NJL model.

(b) The other issue related to the pseudoscalar – axial-vector mixing is that it affects the hadronic amplitudes [55]. In particular, it is well-known [35], [57] that contributions due to πa_1-mixing to the anomalous $\gamma \to \pi^0\pi^+\pi^-$ amplitude violate the low-energy theorem $F^\pi = ef_\pi^2 F^{3\pi}$ [58]–[60], which relates the electromagnetic form factor $F_{\pi^0\to\gamma\gamma} = F^\pi$ with the form factor $F_{\gamma\to\pi^0\pi^+\pi^-} = F^{3\pi}$ both taken at vanishing momenta of mesons. In this thesis we show, for first time, how the surface terms help us to solve this problem. The same strategy we also apply to obtain the rates and spectra of the anomalous $\eta \to \gamma\pi^+\pi^-$ and $\eta' \to \gamma\pi^+\pi^-$decays and clarify the role of πa_1-mixing in these processes.

(c) We discuss for first time the matter of satisfying the Nambu sum rule in the model [43], [44]. It is well known that two Higgs doublets contain eight real fields, three of which are absorbed by gauge $W^\pm$ and Z bosons as a result of the action of the Higgs mechanism. Of the other five fields, two charged fields $h^\pm$ are Nambu partners and, apparently, no problem concerning the observation of the sum rule should arise here. However, three neutral modes χ_1, χ_2, and ϕ_0, entangle the pattern of separating the Nambu partners. As a result, the Nambu sum rule assumes a different form that does not directly associate the masses of the Higgs states with the gap in the fermionic spectrum. We show, for first time, that the cause is related to the global $U(1)_A$ symmetry breaking, for which the't Hooft four-quark interaction is responsible. Since it is suppressed in the leading in $1/N_c$ approximation, the Nambu sum rule is not violated in the limit $N_c \to \infty$.

Scientific novelty

a) We show that, in the effective chiral theory with axial-vector mesons, anomalous Ward identities can be satisfied by taking into account the contributions of surface terms of the triangle quark diagrams, arising due to πa_1-mixing.

b) We show that in the chiral theory with axial-vector mesons and Ward identities fulfilled, there are anomalous diagrams that violate the hypothesis of vector meson dominance.

c) Based on the low energy effective chiral lagrangian we investigate $\eta/\eta' \to \gamma\pi^+\pi^-$ mode by clarifying the role of πa_1mixing mechanism which is completely ignored in the literature. We show for the first time that the parameter $\delta^{(\prime)}$ arises as a result of gauge covariant πa_1 diagonalization and is the residual $U(3)$ breaking effect after applying the Ward identities to the amplitudes of $\eta/\eta' \to \gamma\pi^+\pi^-$ decays.

d) The new interesting physical consequences we have found are related with the anomalous $f_1(1285) \to \gamma\pi^+\pi^-$ and $a_1(1260) \to \gamma\pi^+\pi^-$ decays. In both cases, the new coupling $\bar{q}q\gamma\pi$ not only restores the local gauge symmetry, but also generates a surface contribution to the amplitude. It gives us one of the rare nontrivial field-theoretical examples of how, when calculating the final contributions from single-loop quark diagrams, there arises a surface term whose dimensionless constant cannot be fixed by the theory.

e) We use for first the Schwinger-DeWitt techniques to construct the effective Lagrangian for the Higgs sector of the standard model. As a result, we calculate the spectrum and make numerical estimates for the masses of the composite Higgs states.

The main results of the thesis submitted for defense:

1) The solution of πa_1 mixing problem in the anomalous sector of mesonic interactions: We have shown for the first time that in the effective chiral theory with axial vector mesons, anomalous Ward identities can be satisfied by taking into account the contributions of surface terms of the triangle quark diagrams, arising due to πa_1 mixing.

2) The mechanism of VMD breaking in the low energy meson Lagrangian: We have found for the first time that in the chiral theory with spin-1 mesons, and photons new vertices arise (due to πa_1 diagonalization) that violate the VMD hypothesis, and which are necessary to fulfill the Ward-identities.

3) Two 4-dimensional examples supporting the Jackiw --Rajaraman idea that one-fermion-loop calculations sometimes lead to the finite but theoretically undetermined result: We have found for the first time that a new coupling $\bar{q}q\pi\gamma$, in anomalous $f_1 \to \gamma\pi^+\pi^-$and $a_1 \to \gamma\pi^+\pi^-$decays, not only restores the local gauge symmetry, but also generates a surface contribution to these amplitudes. We show that this contribution gives us one of the rare nontrivial field theoretical examples of how, when calculating the finite contributions from

one-loop quark diagrams, there arises a surface term whose dimensionless constant cannot be fixed by the theory.

4) The role of πa_1 mixing in the anomalous $\eta/\eta' \to \gamma\pi^+\pi^-$decays: We have shown for the first time that parameter $\delta^{(\prime)}$ describing a nonresonant part of the amplitude arises as a result of gauge covariant PA diagonalization and is the residual $U(3)$ breaking effect after applying the Ward identities to the amplitudes of $\eta/\eta' \to \gamma\pi^+\pi^-$ decays.

5) The mass formulas for Higgs states in the composite two-Higgs-doublets model: We have used for the first time the Schwinger - DeWitt techniques to obtain the effective Lagrangian for the Higgs sector of the standard model. From this Lagrangian we have found the mass formulas and made numerical estimates of the composite Higgs states.

6) The Nambu sum rules for Miransky Tanabachi Yamawaki (MTY) model: We have established for the first time the Nambu sum rules in the MTY model and explained the mechanism of their violation as being the axial $U(1)$ symmetry breaking effect.

Approbation of the thesis

The results obtained in the dissertation were reported and discussed at the seminars of the Laboratory of Theoretical Physics, JINR; theoretical department of the Institute of Modern Physics, Chinese Academy of Sciences (Lanzhou, China); on the meeting of the COST Action group CA16201, “PARTICLEFACE 2019” (26 February - 28 February 2019, Coimbra, Portugal); On the Workshop ”QED and QCD Effects in Atomic and Hadron Physics” (31 January - 5 February 2018), Lanzhou, China; on the Workshop ”QCD and Hadron Physics” (10 October - 11 October 2018, Beijing, China); on the Conference "Excited QCD 2020", Krynica Zdrój, Poland (February 2–8, 2020); on the 5th International Conference on Particle Physics and Astrophysics, Moscow (5-9 October)

Confidence level

The approaches and methods used in the dissertation are proven methods of quantum field theory which applied to problems of particle physics: the method of effective Lagrangians and the Fock-Schwinger proper-time method. The dissertation contains a detailed bibliography of the methods used. The reliability of the methods developed in the dissertation and results obtained was ensured by comparing the calculations with the low-energy theorems of QCD, Ward identities, with available alternative approaches, and recent experimental data.

List of publications

1. A. A. Osipov and M. M. Khalifa “Electromagnetic interactions of mesons induced by axial-vector–pseudoscalar mixing” PHYSICAL REVIEW D 98, 036023 (2018).
2. A. A. Osipov, M. M. Khalifa, and B. Hiller “Low-energy theorem for $\gamma \to 3\pi$: Surface terms against πa1 mixing”PHYSICAL REVIEW D 101, 034012 (2020).

3. A. A. Osipov, A. A. Pivovarov, M. K. Volkov, and M. M. Khalifa, “Account for axial vector mesons in the $\eta \to \pi^+ \pi^- \gamma$ and $\eta' \to \pi^+ \pi^- \gamma$ decays” PHYSICAL REVIEW D 101, 094031 (2020).
4. A. A. Osipov and M. M. Khalifa “Catalysis of the $\langle \bar{b}b \rangle$ Condensate in the Composite Higgs Model” JETP Letters, 2019, Vol. 110, No. 6, pp. 387–393.
5. A. A. Osipov and M. M. Khalifa “The Nambu Sum Rule in the Composite Two Higgs Doublet Model” Physics of Particles and Nuclei Letters, 2020, Vol. 17, No. 3, pp. 296–302.
6. A. Osipov, M. M. Khalifa, and B. Hiller “Gauge-covariant diagonalization of πa_1mixing and the resolution of a low energy theorem” Acta Physica Polonica B Proceedings Supplement No 1 Vol. 14 (2021).
7. A. A. Osipov and M. M. Khalifa “Masses of two Higgs doublets within effective theory with four-quark interactions” Journal of Physics: Conference Series, No 1 Vol.1690.(2020) 12075.

Personal contribution of the author

The author of the thesis took part in the formulation of problems, in the discussion of the methods used for their solutions, in the obtaining of the results, and in writing articles. The applicant's contribution to the results of the dissertation is decisive.

The structure and amount of the thesis

The thesis consists of an abstract, introduction, 6 chapters, 8 appendices, and list of 189 references. The full volume of the dissertation is 125 pages, including 13 figures and 1 table.

Content of the work

The introduction substantiates the relevance of the study, describes the strategy and methods used to solve the problems raised, formulates the main results of the thesis, and contains the list of publications made.

CHAPTER-1. This chapter contains material that introduces the basic concepts and methods on which the research is based. Acquaintance with them will allow one to understand the place and range of tasks solved in these theses. This preliminary chapter is also useful for objectively assessing the originality of the ideas underlying the solutions proposed in the following chapters.

CHPTER-2 “Electromagnetic interactions of mesons induced by axial-vector–pseudoscalar mixing” The purpose of this chapter is to study the consequences of the covariant diagonalization in the photon-meson Lagrangian [61]. Our starting point is the NJL model with $SU(2) \times SU(2)$ chiral symmetric four-quark interactions. Here we extend this model by including electromagnetic interactions and show that the covariant diagonalization leads to new electromagnetic vertices where a quark-antiquark pair interacts directly with the photon and the pion. As a result, the theory deviates from the vector meson dominance (VMD) scheme, but possesses the gauge symmetry. To illustrate our theoretical arguments, we consider several examples. The aim is to reveal the specific role of the new electromagnetic vertices induced by the πa_μ-diagonalization. For instance, in the case of the $a_1 \to \pi\gamma$ decay, the results of the old and new approaches are shown to be identical on the mass shell. The $\gamma\pi\pi$ amplitude does not change too.

The anomalous $f_1(1285) \to \gamma\pi^+\pi^-$ decay amplitude is shown to be gauge invariant in both cases, but the results differ. The $a_1(1260) \to \gamma\pi^+\pi^-$ amplitude is not gauge invariant in the conventional approach, but it is invariant in the new version. The latter two processes give a very interesting and rare example for which the surface term of the triangle anomaly cannot be fixed by the Ward identities. Both amplitude contain a free parameter which should be determined only by the experiment.

To avoid the mixing, one usually defines a new axial-vector field through the replacement

$$a'_\mu = a_\mu + km\partial_\mu p\,. \quad (1)$$

We demonstrate that the usual derivative of the pseudoscalar field $\partial_\mu p$ should be replaced by the gauge covariant one ($\mathcal{D}_\mu p = \partial_\mu p - ieA_\mu[\mathcal{Q}, p]$) to conserve the gauge symmetry in the relevant functional integral

$$a'_\mu = a_\mu + km\mathcal{D}_\mu p\,. \quad (2)$$

Numerous specific examples, considered in this chapter, make it possible to verify the consistency of replacements (2).

The covariant derivative contains a direct interaction of a photon with a pseudoscalar meson and a quark-antiquark pair. This brings the theory beyond the generally accepted picture of vector meson dominance. In this section, we explicitly demonstrate that this step leads to a definite physical consequences.

To show this, we consider some electromagnetic processes where novel vertices are involved. The aim of providing these examples is not to offer an exhaustive overview of the possible physical consequences, but rather to demonstrate that such consequences really take place.

Note that the changes are mainly related with a modification of a local replacement of variables in the theory [instead of (1) we use (2)] and, in accord with the Chisholm's theorem [62], [63], should not alter the S-matrix. It is easy to understand why, in spite of this expectation, the results differ. The reason for this is contained in the gauge symmetry requirement. Violating the gauge symmetry, the change (1) leads to the contradiction with the Ward identities and because of that cannot be considered as an equivalent transformation of the theory. Nonetheless, we show that in some cases, the replacement (2) leads to the same result as the replacement (1) [the $\gamma\pi\pi$ vertex], or the results differ by only their off-shell behavior (the $a_1 \to \pi\gamma$ decay).

The real physical consequences we have found are related with the anomalous $f_1(1285) \to \gamma\pi^+\pi^-$ and $a_1(1260) \to \gamma\pi^+\pi^-$ decays. In both cases, the new coupling $\bar{q}q\gamma\pi$ not only restores the local gauge symmetry, but also generates a surface contribution to the amplitude. It gives us one of the rare nontrivial field-theoretical examples of how, when calculating the final contributions from single-loop quark diagrams, there arises a surface term whose dimensionless constant cannot be fixed by the theory. This is one of the main results of this chapter.

CHPTER-3 "Low-energy theorem for $\gamma \to 3\pi$: Surface terms against πa_1 mixing" The Wess-Zumino [64] effective action precisely describes all effects of QCD anomalies in low-energy processes with photons and Goldstone bosons. In particular, this effective action describes the low energy theorem which relates the coupling constants of $\pi^0 \to \gamma\gamma$ and $\gamma \to 3\pi$ amplitudes. In this Chapter [65], [66], we investigate the latter processes by deriving their amplitudes in the framework of the Nambu-Jona- Lasinio (NJL) model with spin-1 states [21]. Then we show how the unwanted contributions due to πa_1 mixing can be suppressed in the $\gamma \to 3\pi$ amplitude. The procedure is based on a careful treatment of the surface terms arising due to the superficial linear divergence of the AVV and AAA triangle graphs (A, axial-vector; V, vector) [67]. One should emphasize that the corresponding low-energy theorem of current-algebra [58] $F^{\pi} = ef_{\pi}^{2}F^{3\pi}$ (which relates the electromagnetic form factor $F_{\pi^0\to\gamma\gamma} = F^{\pi}$ with the form factor $F_{\gamma\to 3\pi} = F^{3\pi}$) can be fulfilled in the NJL model with spin-1 mesons only if there is a deviation from the VMD hypothesis. We come to this conclusion through the gauge covariant treatment of πa_1 mixing [68].

$$a_{\mu} \to a_{\mu} + \frac{\mathcal{D}_{\mu}\pi}{a g_{\rho} f_{\pi}}, \qquad \mathcal{D}_{\mu}\pi = \partial_{\mu}\pi - ieA_{\mu}[\mathcal{Q},\pi] \qquad a = \frac{m_{\rho}^{2}}{g_{\rho}^{2}f_{\pi}^{2}} = 1.84. \tag{3}$$

This modification is relevant for the calculation of $\gamma \to 3\pi$ process, because it yields an additional diagram that contributes to the $\gamma \to 3\pi$ amplitude (see Fig. 1). The $\bar{q}q\gamma\pi$ vertex of this diagram stems from $D_{\mu}\pi$ and induces a deviation of the theory from the complete VMD. The corresponding amplitude is given by

$$\mathcal{M}_{\gamma\to 3\pi} = -\frac{N_c e}{4\pi^2 f_{\pi}^{3}} \epsilon_{\mu}(q)\varepsilon_{\mu\nu\rho\sigma} p_{\nu}^{0}(p^{+} + p^{-})_{\rho} \left(\frac{d_{\sigma}}{4a^{3}}\right) \tag{4}$$

The arbitrary four-vector $d_{\sigma} = b_1 q_{\sigma} + b_2(p^{+} - p^{-})_{\sigma} + b_3(p^{-} + p^{+})_{\sigma}$ contributes to (4) only by its second term b_2. That yields

$$\Delta F^{3\pi} = \frac{N_c e}{12\pi^2 f_{\pi}^{3}}\left(\frac{-3b_2}{2a^{3}}\right). \tag{5}$$

The arbitrary parameter b_2, in the full $\gamma \to 3\pi$ amplitude, can be fixed in such a way that the low-energy theorem $F^{\pi} = ef_{\pi}^{2}F^{3\pi}$ is fulfilled. This gives

$$b_2 = a + \frac{1}{12} = 1.92. \tag{6}$$

Thus, the solution of the πa_1-mixing problem in the $\gamma \to 3\pi$ amplitude is associated with the surface contribution of the anomalous non-VMD diagram shown in Fig.1.

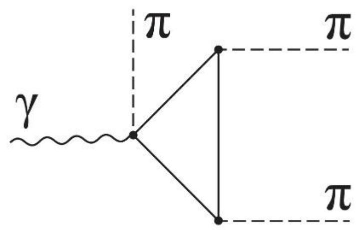

FIG. 1. The quark-loop graph contributing to the $\gamma \to 3\pi$ decay in the NJL model with the covariant πa_1 diagonalization (2). Both single pion lines are the result of πa_1 mixing. The graphs without πa_1 mixing on the single pion lines vanish.

One should stress that a new triangle graph is finite but contains a superficial linear divergence. Because of the linear divergence, shifting the integration momentum in the closed quark loop changes the value of the integral, so that there is an essential ambiguity, d_{σ}, used to satisfy the low-energy theorem $F^{\pi} = ef_{\pi}^{2}F^{3\pi}$. This mechanism is beyond the VMD framework and deserves to be further investigated in the future.

Let us notice that the $\gamma \to 3\pi$ amplitude is already the second example (after the $a_1 \to \gamma\pi^{+}\pi^{-}$ decay) where surface terms associated to an anomalous AAA contribution arising within a gauge covariant description of πa_1 mixing allow us to meet the important symmetry requirements [69]. At the core of both problems has been the non-gauge invariant VAAA box amplitude ($T_{\mu\nu\sigma\lambda}^{F}$

in notations of [60]$^{)}$. In the NJL model with spin-1 mesons this vertex appears also in the context of πa_1 mixing. For instance, in the description of the $a_1 \rightarrow \gamma\pi^+\pi^-$decay this graph arises in the amplitude with two πa_1 transitions, i.e., at the level of two derivatives, and leads to gauge-symmetry breaking. In the $\gamma \rightarrow 3\pi$ amplitude it is an integral element of two types of problematic Feynman diagrams—with two and three πa_1 transitions. Fortunately, the problem, as we just showed, is solved due to the contribution of the surface term of the diagram Fig. 1. Incidentally, it was the inclusion of surface contributions in Ward identities that made it possible to obtain a reliable field-theoretical picture in relating $\gamma \rightarrow 3\pi$ to $\pi^0 \rightarrow \gamma\gamma$ through Ward identities. The Ward identities involved there are in fact not anomalous. However, the surface term, which is normally dropped in standard applications of current algebra, cannot be dropped in the correct Ward identities for $\gamma \rightarrow 3\pi$, where we must keep terms to third order in momentum.

In the end of this Chapter, we stress that one can satisfy the integrability conditions for the anomalous Ward identities by constructing a particular effective Lagrangian that describes exactly the anomalous interactions between the low-lying pseudoscalar mesons and their vector and axial-vector partners (see, for instance) [37]. Our studies apparently indicate that the underlying quark structure of the bosonized NJL Lagrangian allows for a natural implementation of the anomalous Ward identities based on the appropriate classification of the surface terms that emerge in the calculations of the anomalous one-quark-loop triangle diagrams. The realization of this program in the form of some subtraction procedure would make our approach more general and would help to shed light on the role of surface terms in the construction of the anomalous effective action.

CHPTER-4 "Account for axial vector mesons in the $\eta \rightarrow \pi^-\pi^+\gamma$ and $\eta' \rightarrow \pi^-\pi^+\gamma$ decays" The purpose of this Chapter is to demonstrate that Pseoduscalar Axial-Vector (PA) mixing affects a non-resonant contribution in the anomalous $\eta/\eta' \rightarrow \pi^-\pi^+\gamma$ decays of eta mesons, i.e., the box anomaly [70]. The selection of processes is not accidental. These decays are reasonably well studied experimentally and make it possible to measure the magnitude of the non-resonant contribution [71]; moreover, they are sensitive to the flavor symmetry breaking. These modes have been also extensively investigated in the framework of the chiral perturbation theory ($ChPT$) [72], and in different models based on specific chiral Lagrangians with vector mesons [73]. Despite the great work done, a violation of flavor symmetry through the mechanism of eliminating PA mixing has not yet been addressed in the literature. This can be partly explained by the problem that arises when considering axial-vector mesons. After elimination of the PA mixing, some meson amplitudes receive additional contributions that violate a number of low-energy theorems of current algebra and PCAC (partially conserved axial-vector current) hypothesis [74]. Owing to this problem, accounting for contributions from axial-vector mesons to the $\eta/\eta' \rightarrow \pi^-\pi^+\gamma$ amplitudes is not a straightforward issue.

At the one-quark-loop level the $\eta \rightarrow \pi^-\pi^+\gamma$ amplitude receives contributions from the box, ρ-exchange, and the new triangle (induced by considering gauge covariant derivative in the theory) diagrams. In this chapter, we obtain the contributions of these new triangle diagrams to the amplitudes and discuss the physical content of these contributions for the decays under consideration.

The essential ambiguity related with surface terms of anomalous triangle diagrams has been used to satisfy the low-energy theorems for the $\eta/\eta' \rightarrow \pi^-\pi^+\gamma$ decays in the NJL model with spin-1 states. As a consequence, we have found that chiral anomaly not only determines the transition strength prefactor of these amplitudes $A_0^{(\prime)}$, but also explains the origin of the $U(3)$ breaking corrections accumulated in the non-resonant part described by the parameters δ'. The latter quantities have been extracted from the experiment. However their values are quite sensitive to the shape of the spectrum including the region where we do not have high quality data yet. The future more accurate data may significantly influence the integrated rate and therefore the values of $\delta^{(\prime)}$. For this reason, it seems reasonable to establish a solid framework for theoretical calculation of these parameters. The main result of this Chapter is that it suggests a new important contribution for such calculations. We show that $\delta^{(\prime)}$ arise as a result of gauge covariant PA diagonalization and are the residual $U(3)$ breaking effect after applying the Ward identities to the amplitudes of $\eta/\eta' \to \pi^-\pi^+\gamma$ decays.

An important result of this Chapter is also the fact that we were able to extend the known approach to the description of more complex processes with η, η' mesons [69] The nontrivial nature of the problem led to an interesting result—an explanation of the appearance of the parameters $\delta^{(\prime)}$ in the amplitudes of these decays [75].

In addition to the already mentioned applications of the result obtained here, we note the emerging new strategy for extracting the 1^{++} nonet singlet-octet mixing angle from the $\eta/\eta' \to \pi^-\pi^+\gamma$ decays. The extraction of $f_1\,(1285) - f_1(1420)$ mixing angle θ_{f_1} is associated with the processes directly related to the radiative decays of these mesons, or with the use of the Gell-Mann-Okubo mass formula together with the $K_1\,(1270) - K_1(1400)$ mixing angle [76]. It seems one can try to extract $\theta_{f_1}$from the $\eta/\eta' \to \pi^-\pi^+\gamma$ decays too. The reason is that the parameters $\delta^{(\prime)}$ most likely depend on this angle through the mechanism of $\eta,\eta' - f_1\,(1285), f_1(1420)$ mixings.

CHPTER-5 "Catalysis of the$\langle\bar{b}b\rangle$ Condensate in the Composite Higgs Model" Using the effective potential of the theory obtained in the leading $1/N_c$ approximation, we demonstrate that four-quark interactions responsible for the formation of the composite $(\bar{t}t)$ Higgs boson catalyze $\bar{b}b$ condensation [77]. In this case, the 't Hooft four-quark interaction (at any arbitrarily small coupling constant) induces the precipitation of the condensate and the generation of the mass of the b quark. This physical phenomenon is roughly similar to the magnetic catalysis effect. The other issue raised in this chapter are physical consequences of the Nambu sum rule [78], which relates the spectrum of boson modes in a certain channel to the dominant mass of the quark condensate m_f: $\sum m_{boson}^2 = 4m_f^2$. Nambu first applied this rule to superfluid ^{3}He-B and to Bardeen–Cooper–Schrieffer superconductivity and later in top-condensation model. We show that the sum rule is also valid in the model proposed by Miransky, Tanabashi, and Yamawaki (MTY) [43], [44] and allows predicting the masses of partners of the ground Higgs state.

Starting from some approximation to the effective potential of MTY model, we analyze the vacuum structure for the eight bosons and the corresponding gap equations, finally we present the mass formulas for the boson modes as follow:

$$m_{\tilde{\sigma}_0}^2 = 2(m_t^2 + m_b^2), \quad (7)$$

$$m_{\tilde{\pi}_i}^2 = 0, \quad (8)$$

$$m_{\tilde{\sigma}_i}^2 = m_{\tilde{\pi}_0}^2 = \frac{2g_2}{g^2 I_1(m^2)}\left(\frac{m_t^2 + m_b^2}{m_t m_b}\right), \quad (9)$$

where the factor

$$I_1(m^2) = \frac{N_c}{4\pi^2}\left[ln\left(1+\frac{\Lambda^2}{m^2}\right) - \frac{\Lambda^2}{\Lambda^2+m^2}\right]. \tag{10}$$

The spectrum of scalar excitations can be easily found in the form

$$m^2_{\tilde{\sigma}_0} = 2\mathrm{v}^2\lambda, \qquad \mathrm{v} = \sqrt{(m_t^2+m_b^2)\, I_1/2} \simeq 246 \text{ GeV} \tag{11}$$

$$m^2_{\tilde{\pi}_i} = 0, \tag{12}$$

$$m^2_{\tilde{\pi}_0} = m^2_{\tilde{\sigma}_i} = M^2_{H_2} + \lambda \mathrm{v}^2 = \frac{4\sqrt{\mathrm{g}_2^2+\mathrm{g}_3^2}}{\mathrm{g}^2 Z_H}. \tag{13}$$

where v is the vacuum expectation of the Higgs field. Using this value and the known masses of heavy quarks $m_t = 173\, GeV$ and $m_b = 4.18\, GeV$, we obtain the cutoff parameter $\Lambda = 3 \times 10^{10}\ GeV$, which characterizes the scale at which the $\overline{t}t$ condensate is formed. The model under consideration allows the representation of this result in terms of quark masses. Indeed,

$$\lambda \mathrm{v}^2 = \frac{2\mathrm{v}^2}{Z_H} = \frac{\mathrm{v}^2}{Z_H}(\lambda_t^2+\lambda_b^2) = m_t^2 + m_b^2. \tag{14}$$

The used approximation is insufficient to numerically estimate the mass of the state H_2. Indeed, formula (9) is applicable in the range $0 \le m_{H_2} \le \infty$. The lower bound of this range is reached at the phase transition point, where this state becomes a Goldstone mode, and the upper bound is reached at the constraint on the coupling constant $\mathrm{g}^2 = 0$.

Below, we discuss an alternative approach based on the Nambu sum rules [53], [54], which in this case has the form

$$m^2_{\tilde{\sigma}_0} + m^2_{\tilde{\sigma}_3} = 4m_t^2, \tag{15}$$

$$m^2_{\tilde{\sigma}^+} + m^2_{\tilde{\sigma}^-} = 4m_t^2. \tag{16}$$

According to Eq. (15), $m_{\tilde{\sigma}_i} = \sqrt{2}m_t$ because $m_{\tilde{\sigma}_0} = \sqrt{2}m_t$; i.e., the spectrum becomes almost degenerate (with the accuracy to the nonzero mass of the b quark).

A similar conclusion was made in [53], [54]. Using a deep relation between spontaneous symmetry breaking in superfluid ${}^3_1He$-A (which occurs through the same scenario, i.e., through the same gauge symmetry group as in the Standard Model) and Nambu sum rules, the authors of hypothesized the existence of two charged Higgs particles with the mass $\sqrt{2}m_t = 245\, GeV$ and a neutral partner of the standard Higgs boson with a mass of about $325\ GeV$. The only difference of our consideration is the conclusion that these states are degenerate in mass. Thus, it can be concluded that the result $m^2_{\tilde{\sigma}_0} \simeq \sqrt{2}m_t$ does not contradict the Nambu sum rule but requires further study, which we primarily associate with the development of more rigorous method which is presented in the next chapter.

CHPTER-6 "The Nambu Sum Rule in the Composite Two Higgs Doublet Model" In this chapter, we derive the Nambu sum rule in the MTY model [79], [80]. It is well known that two Higgs doublets contain eight real fields, three of which are absorbed by gauge $W^{\pm}$ and Z bosons as a result of the action of the Higgs mechanism. Of the other five fields, two charged fields $h^{\pm}$ are Nambu partners and, apparently, no problem concerning the observation of the sum rule should arise here. However, three neutral modes, χ_1, χ_2 and φ_0, entangle the pattern of separating the Nambu partners. As a result, the Nambu sum rule assumes a different form that does not directly associate the masses of the Higgs states with the gap in the fermionic spectrum. We show that the cause is related to the global $U(1)_A$ symmetry breaking, for which the't Hooft four-quark interaction is responsible. Since it is suppressed in the leading in $1/N_c$ approximation, the Nambu sum rule is not violated in the $N_c \to \infty$ limit.

The Schwinger–DeWitt method [81] has been used for the first time to calculate the spectrum of the MTY model. We show that the method allows obtaining to leading order in $1/N_c$ a most general expression for the effective action of the MTY model. Our approach results in phenomenologically acceptable values of both the mass of heavy quarks and the mass of the standard Higgs state. This solves the basic phenomenological problem of the top condensation model at least to leading order in $1/N_c$.

Starting from the four-quark interactions of MTY model and using Schwinger–DeWitt proper time method to obtain the action of the induced effective low-energy theory, we find the Lagrangian associated with Higgs doublet mode from which we extract the following mass formulas:

$$m_{\chi_1}^2 = 4m^2 + \frac{2g_2}{\bar{g}^2\bar{C}_2}\left(\frac{1}{\cos 2\theta} - \frac{1}{\cos 2\theta'}\right), \tag{17}$$

$$m_{\chi_2}^2 = 4m^2 + \frac{2g_2}{\bar{g}^2\bar{C}_2}\left(\frac{1}{\cos 2\theta} + \frac{1}{\cos 2\theta'}\right), \tag{18}$$

$$m_{\varphi_0}^2 = \left(\frac{4g_2}{\bar{g}^2\bar{C}_2\cos 2\theta}\right), \tag{19}$$

$$m_{h^\pm}^2 = \left(\frac{4g_3}{\bar{g}^2\bar{C}_2\sin 2\theta}\right), \tag{20}$$

$$m_{\varphi_i}^2 = 0. \tag{21}$$

It follows that, of the eight spinless states of the theory, three are massless Goldstone modes that are absorbed by the gauge fields (the Higgs mechanism). As can be easily seen from above formulae, the other five states satisfy the sum rule as

$$m_{\chi_1}^2 + m_{\chi_2}^2 + m_{\varphi_0}^2 = \left(\frac{8g_3}{\bar{g}^2\bar{C}_2\sin 2\theta}\right), \tag{22}$$

$$m_{h^+}^2 + m_{h^-}^2 = \left(\frac{8g_3}{\bar{g}^2\bar{C}_2\sin 2\theta}\right). \tag{23}$$

This result differs somewhat from the Nambu sum rule. Although the sum of the squared masses of the neutral modes and the analogous sum for the charged modes equal the same expression, its value does not coincide with $4m_t^2$, as it is required by the Nambu sum rule. Furthermore, instead of two Nambu partners, the first expression contains contributions of three states, which also deviates this result from the standard rule. What are the reasons for that? To answer this question, we write two other relations that are also a consequence of mass formulae (17)–(21) as follows:

$$m_{\chi_1}^2 + m_{\chi_2}^2 = m_{\varphi_0}^2 + 8m^2, \tag{24}$$

$$m_{h^+}^2 + m_{h^-}^2 = 2m_{\varphi_0}^2 + 8m^2. \tag{25}$$

where $2m^2 = m_t^2 + m_b^2$. It can be seen from this that a mass of meson φ_0 that is different from zero prevents the Nambu sum rule from being satisfied.

The numerical estimation of the Higgs state masses in formulae (17)–(21) can be given as

$$m_{\chi_1}^2 = \frac{2\mathrm{m}^2}{a-1}(2a - 1 - \Delta), \tag{26}$$

$$m_{\chi_2}^2 = \frac{2\mathrm{m}^2}{a-1}(2a - 1 + \Delta), \tag{27}$$

$$m_{\varphi_0}^2 = \frac{4\mathrm{m}^2}{a-1}, \tag{28}$$

$$m_{h^\pm}^2 = \frac{4\mathrm{m}^2 a}{a-1}, \tag{29}$$

where $\frac{g_3}{g_2} = a\tan 2\theta$ and $\Delta = \sqrt{cos^2 2\theta + (3-2a)^2 sin^2 2\theta}$.

If parameter a is fixed by the known value of the mass of the standard Higgs state $m_{\chi_1} = 125\ GeV \rightarrow a = 4.84$, the above formulae yield the following numerical estimates: $m_{\chi_2} = 346$ GeV, $m_{h^\pm} = 275 GeV$, and $m_{\varphi_0} = 125\ GeV$. The fact that the mass $m_{\chi_1} \simeq m_{\varphi_0}$shows that $\Delta = 2a - 3$; i.e.,the angle $\theta \simeq \pi/4$. To make the final conclusion about the reasonableness of these estimates, the renormalization group approach must be applied, the findings of which are set forth in a separate study.

These calculations show that the principles of symmetry play a decisive role in the study of the Nambu sum rule, and an explicit violation of symmetry can lead to a deviation from this rule. The chiral $U(2)_V \times U(2)_A$ symmetric meson theory based on four-quark interactions provides a well-known example. Its spectrum is comprised of eight modes, viz., four pseudoscalar π_i and η modes and four scalar δ_i and σ modes with their mass being equal to $m_{\pi_i} = m_\eta = 0$ and to $m_{\delta_i} = m_\sigma = 2M$, where M is the gap in the fermionic spectrum. Four pairs of the Nambu partners can be differentiated, for each of the pairs the sum rule being satisfied as

$$m_{\delta_i}^2 + m_{\pi_i}^2 = 4M^2, \;\; m_\sigma^2 + m_\eta^2 = 4M^2. \tag{30}$$

To eliminate the degeneracy between $SU(2)$ triplets π_i and δ_i and the corresponding singlet states η and σ, the four-quark interaction, the ‘t Hooft determinant, which breaks the axial symmetry, is usually added. As a result η meson acquires mass $m_\eta = k \neq 0$ and the mass of isotopic scalar triplet δ_i becomes higher than the mass of singlet state σ: $m_{\delta_i}^2 = m_\sigma^2 + k^2$ the mass of fields π_i and σ do not change in this case. This affects the Nambu sum rule, which takes the form

$$\begin{gathered} m_{\delta_i}^2 + m_{\pi_i}^2 = 4M^2 + k^2, \\ m_\sigma^2 + m_\eta^2 = 4M^2 + k^2. \end{gathered} \tag{31}$$

We can see that this greatly resembles the picture observed in the top condensation model considered above; namely, the axial $U(1)_A$ symmetry breaking results in an additional contribution to the right term of the Nambu sum rule associated with the presence in the anomalous interaction theory of the Goldstone boson responsible for the mass of $U(1)_A$.

Can we assert that we are dealing here with a violation of the Nambu sum rule? Of course, not. It is well known that in quantum chromodynamics the anomaly vanishes at $N_c \rightarrow \infty$; i.e., it is absent in the leading order in $1/N_c$. Consequently, the Nambu sum rule is satisfied to this approximation. The constant of the four-quark ‘t Hooft interactions G_H behaves at high N_c values as $G_H \sim 1/N_c^{N_f}$, where N_f is the number of the quark flavors with the constant of the fourquark interactions of the

NJL model, changing as $G_H \sim 1/N_c$. Hence it follows that the squared mass of the η meson is of th $m_\eta^2 \sim 1/N_c$ order [82] and, consequently, we are dealing with the correction to the Nambu sum rule, the $1/N_c$ correction of which changes the right term of the relation in question, but only in the next to leading order in $1/N_c$.

The aforementioned also holds true for the model of [43], [44]; therefore, our result (22)–(23) is a natural manifestation of the anomaly and a correction of 26% seems to be quite a reasonable value for the contribution of the next to leading order in $1/N_c$.

The numerical values obtained here for the mass of the Higgs states are in agreement with that obtained in [54]. The difference is not large. Thus, for the mass of the χ_2 state, the value $m_{\chi_2} = 325$ GeV was obtained there and $m_{h^\pm} = 245$ GeV was obtained for the charged particles. These values are slightly lower than our estimates here; this is explained, however, by the $U(1)_A$ anomaly, which increases the right term of the Nambu sum rule and, consequently, the mass of the particles of the second Higgs doublet. The novelty is the presence in the spectrum of the "electroweak φ_0 axion" with a mass that practically coincides with the mass of the Higgs ground state. Although the present Higgs data are consistent with the Standard Model, one cannot yet rule out the presence of a mass degenerate scalar state at 125 GeV. Detailed phenomenological analysis will enable us to gain insight into the future of this prediction of the two Higgs doublet model.

Appendices

At the end of the thesis, we present our appendices. The first two appendices contain useful formulae as well as the heat kernel method for deriving the Seeley-DeWitt coefficients which assist in constructing the effective meson Lagrangian of our approach. The rest of the appendices contain essential mathematical detailed derivations which make the calculations of the spectrum, based on the two Higgs doublet model, to the reader, more transparent.

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